

## 05. PROPORTIONALITY

### PRICE AND SIZE

It is evident that the price is directly proportional to the amount of the commodity.

The price is always linked to the weight, the length,...

We are going to use some information to check if the proportionality is always the rule to value an item.

As you would realize this is a flyer about photography.

What do you think about the price, is it proportional to the side of the photo?

If the answer is negative, what would the price be for the 13 x 18 in a proportional sense?

**muriam morgado**  
ESTUDIO DE FOTOGRAFIA  
Avda. de París nº8 - Telf: 927231138  
24 Fotos 10x15.....6,78 €.  
24 Fotos 13x18.....8,64 €.  
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### PRICE AND RADIUS

Here are three different pizzas. And here are the different prices:

Individual \$3.4

Medium \$8.9

Family \$9.9

You have to do a research about the proportionality.

If you take the personal size as reference, what would the prices be for the others?



## 1. RATIO AND PROPORTION

Division is the best operation to compare two quantities. It tells us how great is one quantity in respect to another.

For example, Spain has 40 millions people and Portugal 20 millions; so Spain has double the population of Portugal because  $40/20 = 2$

Mary earns €300 a week in Movistar and Antonio €200; so Mary receives 1.5 times more salary than Antonio per week because  $300/200 = 1.5$

### RATIO

A ratio is a division of two quantities 'a' and 'b'. That is  $a / b$  or  $a : b$  (read as 'a' to 'b'). Ratio indicates how many times one quantity is greater or smaller than the other. This concept is to compare two like quantities –two lengths for instance- or two unlike quantities –one length and one time-.

Ex:  $\frac{2m}{1m} = 2$ . The numerator is twice the denominator. Like quantities. The ratio doesn't have unit.

Ex:  $\frac{100Km}{2hs}$ . Unlike quantities. This ratio has unit. This is the speed of a car, for example.

Ex: A map scale is 1:150. What does it mean?

### PROPORTION

Proportion is the equality of two ratios. That is  $\frac{a}{b} = \frac{c}{d}$ .

$$\frac{100Km}{2hs} = \frac{200km}{4hs}$$

This is a proportion. That is, the two data are proportional, the two cars travelled at the same speed. The result of the division is the constant of proportionality.

$$\frac{100Km}{2hs} = \frac{200km}{4hs} = 50km/h. \text{ The cars have a common speed.}$$

### ***Fundamental property for a proportion***

Product of extremes = Product of means.

Quantity 1	a		c
Quantity 2	b		d
	$\frac{a}{b} = \frac{c}{d}$ or $a \cdot d = b \cdot c$		

## 2. DIRECT PROPORTIONALITY

### MAGNITUDES IN DIRECT PROPORTION

Two magnitudes whose quotient is constant are directly proportional.

That is, they form a proportion; the increase in one of them means an increase in the other.

$$\frac{\text{Magnitude 1}}{\text{Magnitude 2}} = \text{constant}$$

The constant is the constant of proportionality.

#### Examples

The height and the shadow.

$$\frac{\text{Height}}{\text{Shadow}} = \frac{10m}{30m} = \frac{2m}{6m} = \dots$$

The taller the object is the longer the shadow cast.

The weight and the price of something.

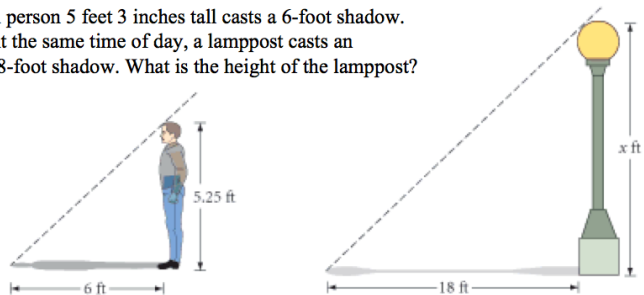
$$\frac{\text{Price(€)}}{\text{Weight(kg)}} = \frac{2€}{1kg} = \frac{4€}{2kg} = \dots$$

The heavier the object is the more expensive the object is.

The price of a product and the tax to pay.

$$\frac{\text{Price(€)}}{\text{Taxes(€)}} = \frac{100€}{21€} = \frac{200€}{42€} = \dots$$

A person 5 feet 3 inches tall casts a 6-foot shadow. At the same time of day, a lamppost casts an 18-foot shadow. What is the height of the lamppost?



### PROBLEMS OF DIRECT PROPORTIONALITY

We can solve these problems in two different ways:

#### Calculation of the constant of proportionality

We calculate the value of a single unit.

Example: If 4 kg of potatoes cost €6, what is the price of 9kg?

### ***Rule of three direct***

It is based on the fundamental property. Product of extremes = Product of Means.

Example: If we have to pay 21% of V.A.T. for a \$250 product; how much V.A.T. do we have to pay for it?

## **3. INVERSE PROPORTIONALITY**

### **MAGNITUDES IN INVERSE PROPORTION**

Two magnitudes are in inverse proportion if the product of every related value is constant, and consequently, the increase of one of them is equal to the decrease of the other.

$$\text{Magnitude 1} \cdot \text{magnitude 2} = \text{constant}$$

This is the constant of inverse proportionality.

### ***Examples***

Number of workers and individual working time.  
Continue with another example.

$$\frac{\text{Number of workers}}{\text{Individual working time}} = \frac{8}{2} = \frac{4}{4} = -$$

The more workers the less time.



Number of participants buying a gift and their contribution. Continue with another example.

$$\frac{\text{Number of participants}}{\text{Individual contribution}} = \frac{10}{6} = \frac{5}{12} = -$$



### ***Fundamental property***

The product of the related values is constant.

Example: 10 friends give €4 each to buy a present for a friend, what is the constant of proportionality?

## PROBLEMS OF INVERSE PROPORTIONALITY

We can solve these problems in two different ways

1. Calculation of the constant of proportionality. We calculate the number of units we need.

Example: 10 friends give €4 each to buy a present for a friend, how much would 8 friends have to give to buy the same present?

2. Rule of three inverse.

Example: 20 men spend 6 days to clean a village, how long does it take 30 men?

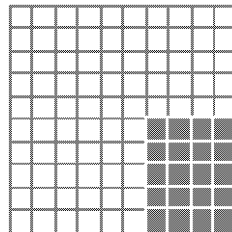
## 4. PERCENTAGES

It is a ratio whose denominator is 100. It is denoted by a % -percent- or  $a/100$ .

For example:

$$54\% = \frac{54}{100}; \quad 25\% = \frac{25}{100}; \quad 7\% = \frac{7}{100}; \quad 120\% = \frac{120}{100}$$

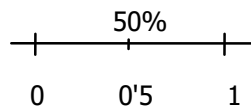
The shaded part represents 20%. The square is divided into 100 parts and this area has 20 parts.



To work out the decimal value of a percentage you only need to .

In this case we get the part over one.

$$50\% = 0.5$$



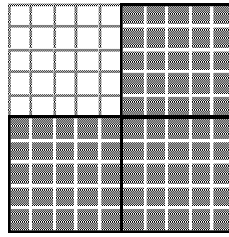
Get the decimal value for the following percentages:

$$25\% = \frac{25}{100} = 0.25; \quad 7\% = \frac{7}{100} = \quad ; \quad 120\% =$$

We can obtain the simplified fraction too:

$$25\% = \frac{25}{100} = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$$

$$75\% =$$



$$\frac{3}{4} = 75\%$$

### **RULE TO CALCULATE A PERCENTAGE**

We can do a direct rule of three.

### **PERCENTAGE INCREASE AND DECREASE**

We consider three questions:

1. The initial quantity. That is the value without the percentage.
2. The final quantity. That is the value with the percentage.
3. The variation index. That is 1 plus or minus the percentage.

#### ***Rule of calculation***

Final quantity = Initial quantity · Variation index

Initial quantity = Final quantity : Variation index

Example: If we have to pay 21% of V.A.T. for a \$250 product; how much tax do we have to pay for it? Calculate it on this way.

## 5. COMPOSITE PROPORTIONALITY

### ***Composite proportionality***

It is the proportionality with more than two proportional magnitudes.

### ***Rule of composite proportionality***

We can solve these problems:

1. Reducing to the unit.
2. By relating the magnitude with the unknown value according to simple rules.

For example,

8 painters painted 4000 m<sup>2</sup> in 20 days. How many days would 10 painters need to paint 6000 m<sup>2</sup>?

To solve the problem we proceed in the following way:

- 1º. We are going to put the unknown variable in the middle.

nº painters	Time (days)	Area (m <sup>2</sup> )
8	20	4000
10	x	6000

- 2º We establish the kind of dependence between it and the others.

I		D
nº painters	Time (days)	Area (m <sup>2</sup> )
8	20	4000
10	x	6000

- 3º We operate according to the corresponding rules.

$$8 \cdot 20 \cdot 6000 = 10 \cdot x \cdot 4000$$

### Example 2:

Reading 2 hours a day I took 10 days to read a 1000 pages book. How long does it take to read a 3000 pages book reading 4 hours a day?

### ***Bank interest***

The simple formula that works for this question involves the following concepts:

- Principal (P): The amount of money deposited in a bank in order to produce a profit or the loan that a bank lends us.
- Interest (I): The amount of money that is produced by the principal during a time or the money that I have to pay for the loan.
- Time (t): The period of time that the money is in the bank or I have the loan.
- Rate (r): The percentage that the bank offers to the customer.
- The formula that relates these terms is:

$$I = P \cdot r \cdot t$$

### ***Example***

If I invest €480 in a bank that gives me an annual rate of 3.5%. What interest will I get two years and a half later?

### ***Useful vocabulary***

Investment: The money put in a bank to make a profit.

Loan: A sum of money which is borrowed from a bank.

Principal: An amount of money which someone has invested in a bank.

Profit or interest: Money which is earned in trade or business.

Rate: The amount of interest that a bank charges.

### **VÍDEOS**

#### ***Ratios:***

<http://goo.gl/c6LFN7>



## PROBLEMS AND EXERCISES

### 1. *Ratio and proportion*

1. Calculate the missing value on these proportions: a)  $\frac{3,2}{5,8} = \frac{4,8}{x}$  b)  $\frac{0,4}{x} = \frac{x}{0,9}$

2. Calculate the unknown term in the following proportions:

a)  $\frac{-3}{5} = \frac{18}{x}$ ; b)  $\frac{14}{35} = \frac{x}{90}$ ; c)  $\frac{2}{x} = \frac{x}{50}$

3. Calculate the unknown term in the following proportions:

a)  $\frac{3}{12} = \frac{5}{x}$ ; b)  $\frac{28}{70} = \frac{x}{90}$ ; c)  $\frac{3}{x} = \frac{x}{75}$

4. Calculate the unknown term in the following proportions:

a)  $\frac{4}{10} = \frac{x}{60}$  b)  $\frac{9}{12} = \frac{12}{x}$  c)  $\frac{8}{32} = \frac{2}{x}$  d)  $\frac{3}{x} = \frac{x}{12}$  e)  $\frac{x}{6} = \frac{24}{x}$

### 2. *Direct proportionality*

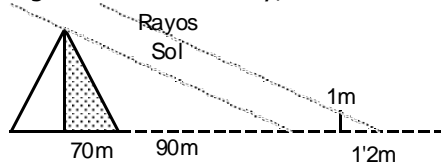
5. Anne buys 5 kg of potatoes at the market. If 2 kg cost \$ 0,80, how much does Anne pay?

6. A clock gains 4 minutes every 28 h. How long does it gain every week?

7. On a piece of paper 20cmx30cm we want to draw a building plan whose dimensions are 150m x 400 m.

What scale would you use?

8. Through the shadow study, how tall is the pyramid?



9. Four members of a family spend €7200 every 6 months. How many people could live 9 months with €21600?

10. A loses gains 2 minutes every 14 h. How long does it lose every week?

11. A room is 20 m<sup>2</sup> and there are 12 units in the radiator. How many units do we need to put in a 4m<sup>2</sup> bathroom?

12. A 250g packet of chips is €2.15 and a 900g packet is €7.20.

- Is the second cheaper per gram than the first?
- What is the constant of proportionality for each?
- What are the savings per kilo choosing the second type of package?

13. A 300 km trip by train was €25. How much will a 450 km trip be?

- 14.** Astronomic unit is the distance between the Earth and the Sun (150 million kilometres). Calculate with the calculator the a.u. for each planet according to the next chart.

Planeta	Milones km	u. a.	Planeta	Milones km	u. a.
Mercurio	58		Jupiter	780	
Venus	108		Saturno	1430	
Tierra	150		Urano	2870	
Marte	228		Neptuno	4500	
Asteroides	400		Plutón	5910	

- 15.** A fish farmer calculates the number of fish in this way:  
He takes out 300 fishes from the pool and makes a spot on them giving them back to the pool again. A day later he takes out 300 fishes again and notices there are only 15 fishes with the spot.

How many fishes are there in the pool approximately?

### 3. Inverse proportion

- 16.** 4 taps can fill a tank in 70 minutes. How long does it take to fill the tank using 7 taps?
- 17.** 6 taps can fill a tank in 60 minutes. How long does it take to fill the tank with 15 taps?
- 18.** Is there proportionality among these variables according to the data? What kind of proportionality is there and what is the constant of proportionality?

x	y
2	10
5	4
20	1

- 19.** 10 taps fill a water tank in 4 hours.  
a) How long does it take to fill it with 8 taps?  
b) If it takes 1 h to fill it, how many taps are working?
- 20.** A container is full of 1200 bags of potatoes 4 kg each. If the bag were 5 kg each, how many bags do we need to fill the container?

### 4. Percentages

- 21.** Of the 800 students at a school, 600 attend a trip. What is the percentage of students who remain at the school?
- 22.** A greengrocer sold 140 Kg of apples last year. But this year he sold a 20% more. How many kg has he sold this year?
- 23.** The fever killed 22% of animals on a farm. Today there are 273 animals, how many animals were there before the fever?
- 24.** The price of a computer is \$1200. How much does a man pay in total if it has a 16% discount?
- 25.** A hotel has 50 rooms and 35 of them are occupied. What percentage of the hotel is occupied?
- 26.** A population has increased from 75000 people to 75820 people in a year. What is the increased ratio in %?
- 27.** We heard the RPI (Retail Price Index) has increased 2.5% last year.  
If I spent €15000 last year, what could my new expenses be this year?
- 28.** Change the following proportions into %:  
a. 4‰; b. 40‰; c. 400‰; d. 3 por 1; e. 0'3; f. 0'03

- 29.** Change the following proportions into ‰:  
a. 3%; b. 302%; c. 300%; d. 4 por 1; e. 0'4; f. 0,04
- 30.** Change the following proportions into a decimal form:  
a. 5%; b. 50%; c. 500%; d. 5‰; e. 60‰; f. 600 ‰

**5. *Composite proportionality***

- 31.** 5 workers working 6 hours a day build a wall in 2 days. How many days do 4 workers take working 7 hours a day?
- 32.** 35 workers take 16 days to make a fence. How many days will it take 28 workers to raise another?
- 33.** 5 workers, working 8 hours a day, do a job in 15 days. How many hours a day would 3 workers have to spend to finish the same work in 20 days?
- 34.** You walked 5 hours a day for 4 days to do a 68 km trek. If you walk 7 hours a day for 5 days, what distance will you cover?
- 35.** They spent €540 to feed 36 teenagers for 12 days in a Camp. How much would it be to feed 58 teenagers for 26 days?
- 36.** How long does it take a principal of \$2500 at a rate of 4% annual to become \$30000?
- 37.** 9 taps open for 10 hours cost 20 € a day. Work out the expenses of 15 taps open for 12 hours.
- 38.** \$45000 is deposited into a savings account. After three years it totals \$52500. Calculate the rate for this account.
- 39.** At what rate do you have to invest €9000 during 200 days to get €250 of profit?
- 40.** \$50 000 is deposited into a savings account 3.5% of interest per year.  
a) How much will be received after 3 years?  
b) How much will be received after 6 months?  
c) And after 4 months?  
d) After three years the total amount is \$52500. Calculate the rate for this account.
- 41.** A friend told me he has got €250 of benefit from a deposit of €12500 in a bank, what is the given interest?